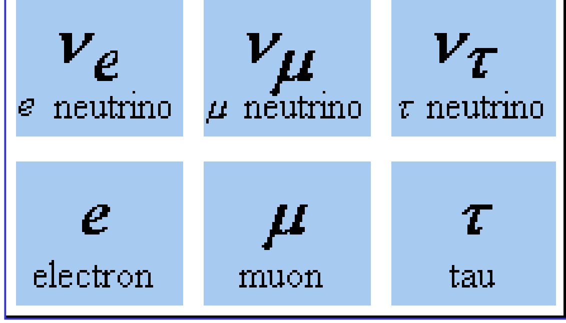
Neutrino Propagation

- Mass and flavour eigenstate
- The oscillation phenomenon
- Survival and transition probability
- Disappearance and appearance experiments
- Kamland and SNO: the oscillation of ν_e and anti ν_e
- Atmospheric and accelerator: the oscillation of ν_μ and ν_τ
- Appendix: neutrinos in vacuum and in matter.

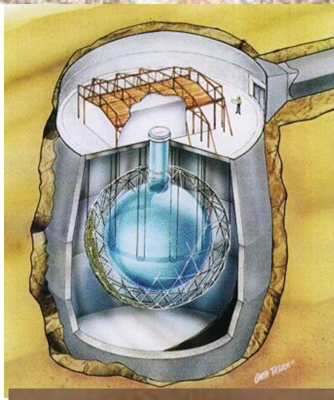
Neutrino Propagation: the vacuum oscillation phenomenon

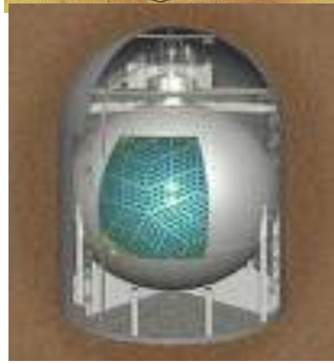


- <u>Are neutrinos of a certain family always the same, once produced, or can they change?</u>
- In 1957 Pontecorvo guessed the possibility of an oscillation mechanism, so that a state, for instance produced as $|\nu_e>$, during its time evolution is allowed to gain a different family component, for example, a muonic component:

$$|\psi>_t = A(t)\,|\nu_e> + B(t)\,|\nu_\mu>$$

- It is clear that such a phenomenon, if real, means a violation of the family lepton number (even if the total lepton number is conserved)
- The interest for this phenomenon, called neutrino oscillation, was born after first results of the solar neutrino experiments, which detected a smaller flux with respect to what was expected from theoretical models: within the sun neutrinos are produced as ν_e , and detectors were sensible to ν_e only. Maybe that a fraction of neutrinos changed flavour during the travel?
- The positive and definite answer to this question has been given in 2001 by SNO experiment (with solar neutrinos) and in the following year by KamLAND (with reactor anti-neutrinos)





Mass and flavour eigenstates (1) *

- Let's consider, for semplicity, only two neutrinos families, ν_e e ν_μ . These two neutrinos have a well defined flavour, by definition, since they are produced together with electrons or muons respectively. On the other hand <u>we can't say that also their mass is well defined.</u>
- Let's consider the space which is generated by $|\nu_e>$ and $|\nu_\mu>$. In the basis $|\nu_e>$ and $|\nu_\mu>$ the family lepton number operators are given by the following matrices:

$$L_e = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad L_\mu = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

- The states $|\nu_e>$ and $|\nu_\mu>$ are consequently eigenvectors of such observables, L_e and L_μ , with eigenvalues 1 and 0 for $|\nu_e>$, 0 and 1 for $|\nu_\mu>$.
- Let's now consider the mass observable M. M is not necessary diagonal in the basis $|\nu_e>$ and $|\nu_\mu>$, so we can't say that $|\nu_e>$ and $|\nu_\mu>$ are mass eigenstates, i.e. they have defined mass.
- In other words, if we call $|\nu_1>$ and $|\nu_2>$ the mass eigenstates, with eigenvalues m_1 and m_2 respectively, they may not coincide with $|\nu_e>$ and $|\nu_\mu>$. In the most general case they will be linear combinations of them.
- If m_1 and m_2 are different, $|\nu_1>$ and $|\nu_2>$ will be orthogonal, since eigenvectors corresponding to different eigenvalues. Consequently they form a new orthogonal basis for the neutrino states space.
- **More properly, in italian we call as "eigenstates" the eigenvectors of a hamiltonian operator, but usually eigenstate and eigenvector are used witohut any differences; in this chapter we will use eigenstate and eigenvector with the same meaning.*

Mass and flavour eigenstates (2)

- Mass (or vacuum) eigenstates (ν_1, ν_2) are in general connected with flavour eigenstates (ν_e, ν_μ) produced by weak interaction through a rotation:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \qquad M \; |\nu_i\rangle = m_i \; |\nu_i\rangle \quad i = 1,2$$

- Namely, state $| \nu_e >$ is a linear superposition of two states with a well defined mass, m_1 ed m_2.

$$| \nu_e > = \cos\theta \, | \nu_1 > + \sin\theta \, | \nu_2 >$$

- If we measure the mass of the $| \nu_e >$ state we will get m_1 and m_2 with probability $\cos^2\theta$ and $\sin^2\theta$ respectively.
- So, we can define the electron neutrino mass as the mean value of the observed results of the mass measurements:

$$m_e = < \nu_e | M | \nu_e > = \cos^2\theta \, m_1 + \sin^2\theta \, m_2 \, .$$

- And analogously for the muon neutrino:

$$m_\mu = < \nu_\mu | M | \nu_\mu > = \sin^2\theta \, m_1 + \cos^2\theta \, m_2 \, .$$

Free neutrino Hamiltonian

2ν

$\hbar = c = 1$

- The free Hamiltonian of a particle moving in vacuum is given by

$$H = (p^2+M^2)^{1/2}$$

- If we fix the momentum p, then mass eigenstates are also hamiltonian eigenstates, with eigenvalues $E_i=(p^2+m_i^2)^{1/2}$ and so, in the basis of the mass aigenstates (ν_1, ν_2):

$$H_\nu = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} = \begin{pmatrix} \sqrt{p^2+m_1^2} & 0 \\ 0 & \sqrt{p^2+m_2^2} \end{pmatrix}$$

- In the ultrarelativistic approximation ($p>>m_i$) one has $E_i \approx p+m_i^2/2p$, so that we can write the hamiltonian in the mass basis as:

$$H_\nu = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \cong \begin{pmatrix} p + \frac{m_1^2}{2p} & 0 \\ 0 & p + \frac{m_2^2}{2p} \end{pmatrix}$$

- Let's remember that the hamiltonian is defined up to a constant, which, in a quantum mechanics picture, brings a time evolution phase equal for all vectors, so that it is unobservable. In this way we can delete the common term "*p*" in the previous expression.
- Similarly we can add a common term $-(m_2^2+ m_1^2)/4p$; if we define $\Delta m^2=(m_2^2- m_1^2)$, the result is, up to constants which are unobservable in the time evolution:

$$H_\nu = \begin{pmatrix} -\frac{\Delta m^2}{4p} & 0 \\ 0 & \frac{\Delta m^2}{4p} \end{pmatrix}$$

The evolution of neutrino in vacuum

- Now that we know the hamiltonian, so we can study the time evolution of neutrinos with fixed mass and of those with fixed flavour.
- About the first ones (hamiltonian eigenstates), the time evolution only gives a change of phase. If we start from states $| \nu_1 >$ and $| \nu_2 >$, their evolution will be:

$$| \nu_1 (t)> \; = \exp(+ i \, \Delta m^2 t / 4p) \, | \nu_1 > \; ; \qquad | \nu_2 (t)> \; = \exp(- i \, \Delta m^2 t / 4p) \, | \nu_2 >$$

- Let's now consider a neutrino with initial flavour $| \nu_e >$. This can be expressed in terms of mass eigenstates as

$$| \nu_e > \; = \cos\theta \, | \nu_1 > + \sin\theta \, | \nu_2 >$$

and the evolved state at time t will be:

$$| \psi > \; = \cos\theta \, | \nu_1 (t)> + \sin\theta \, | \nu_2 (t)>$$
$$= \cos\theta \, \exp(+ i \, \Delta m^2 t / 4p) \, | \nu_1 > + \sin\theta \, \exp(- i \, \Delta m^2 t / 4p) \, | \nu_2 >$$

- We can see that the problem is completely defined by two parameters:
  - the mixing angle θ
  - the difference between neutrinos squared masses $\Delta m^2 = (m_2^2 - m_1^2)$

Survival and transformation probability

- Neutrinos are produced by weak interactions and they are revealed through weak interactions, which project their state in a state with fixed flavour.
- Let's consider a ν_e produced by a weak interaction at time t=0, propagating in vacuum: we want to compute the probability of observing it (that means it is still ν_e) after time t (survival probability):
- As we have already seen, the time evolution of a state which is initially ν_e is given by:

$$|\Psi\rangle_t = e^{-iH_\nu t}\,|\nu_e\rangle = \cos\theta\;\; e^{i\frac{\Delta m^2}{4p}t}\,|\nu_1\rangle + \sin\theta\;\; e^{-i\frac{\Delta m^2}{4p}t}\,|\nu_2\rangle$$

- If we project this vector on $|\nu_e>$ we can compute the survival amplitude:

$$A_{ee} = \langle\nu_e\,|\,\Psi\rangle_t = \cos^2\theta\;\; e^{i\frac{\Delta m^2}{4E}t} + \sin^2\theta\;\; e^{-i\frac{\Delta m^2}{4E}t}$$

and from this the survival probability, so that the neutrino initially born as ν_e is still itself after a time t

$$P_{ee} = |A_{ee}|^2 = 1-\sin^2 2\theta\;\; \sin^2[\Delta m^2 t/(4p)]$$

The probabilty of transformation into another flavour, $P_{e\mu}$, can be obtained by the law of conservation of probability (unitariety of evolution operator), namely P_{ee} + $P_{e\mu}$=1, so that:

$$P_{e\mu} = 1-P_{ee} = \sin^2 2\theta\;\; \sin^2[\Delta m^2 t/(4p)]$$

- Note that we can have oscillation only if $\theta \neq 0$ and $\Delta m^2 \neq 0$
- Note also that both probabilities depend on the neutrino momentum *p*

Neutrino evolution as function of distance: the vacuum oscillation length

- Let's suppose to observe neutrinos at a distance L from prodction. Neutrinos travel with a velocity close to that of light, so we can set v=c; the taken time will consequently be t=L/c, namely t=L if we use c=1.
- We can also observe that neutrinos are ultrarelativistic particles, so that we can approximate their momentum *p* with their energy *E*.
- So we can write the survival and trasformation probability as

$$P_{ee} = |A_{ee}|^2 = 1 - \sin^2 2\theta \ \ \sin^2[\Delta m^2 L/(4E)] \quad \text{and} \quad P_{e\mu} = \sin^2 2\theta \ \ \sin^2[\pi L / L_V]$$

- It is useful to introduce the oscillation length, L_V, defined as the distance at which the oscillation has been completed, and so the phase must be π, i.e.

$$L_v = 4\pi E/\Delta m^2$$

In terms of L_V we get:

$$P_{ee} = |A_{ee}|^2 = 1 - \sin^2 2\theta \ \ \sin^2[\pi L / L_V]$$

- The oscillation length is the most important parameter in oscillation experiments, since in practice there is no oscillation ($P_{ee}=1$, $P_{e\mu}=0$) if $L << L_V$. In order to see the oscillation phenomenon one should have $L \approx L_V$ or greater.
- The oscillation length can be espressed in "ordinary" units, by putting a convenient number of $\hbar$ and c:

$$L_v = \frac{4\pi E\hbar c}{\Delta m^2 c^4} = 2.5m\left(\frac{eV^2}{\Delta m^2}\right)\left(\frac{E}{MeV}\right)$$

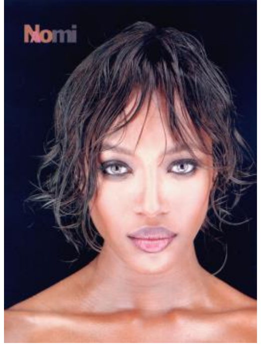
Experimental sensibility

For a physical distance L and neutrino energy E one can explore the neutrino masses differences, such that $L \approx L_V$.

The following table shows the main features of experiments

ν Source	**$\langle E_\nu \rangle$**	**L**	**$\Delta m^2 [eV^2]$**
reactors	~MeV	~10 m	10^{-1}
"long-base" reactors	~MeV	~100 km	10^{-5}
sun	~MeV	~100 Mkm	$10^{-10} - 10^{-11}$
accelerators	~ GeV	~100 m	10
"long-base" accelerators	~GeV	~100 km	10^{-2}
atmospheric	~GeV	~10000km	10^{-4}

Diasappearance and appearance experiments



- In principle, we have two different methods to verify the existence of the oscillation phenomenon:

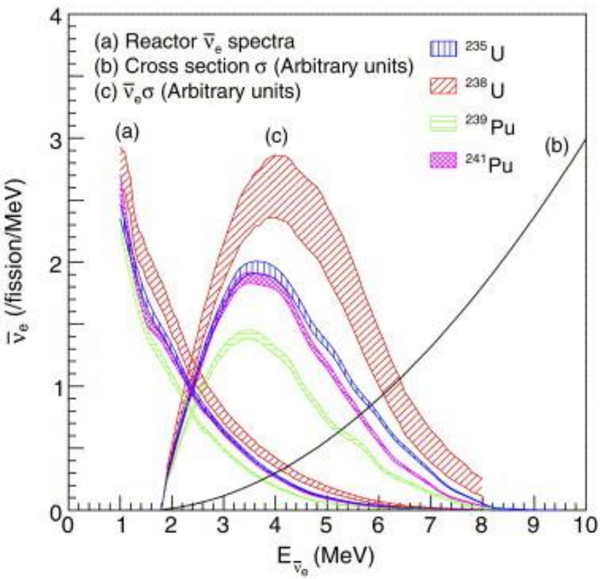
a) <u>disappearance</u>: we produce neutrinos with a given flavour, and we reveal neutrinos with the **same** flavour, after a distance L. If oscillations occur, we'll find a smaller number of neutrinos;

b) <u>appearance</u>: we produce neutrinos with a certain flavour, and then we reveal neutrinos of **different** flavour after a distance L.

- In a disappearance experiment, we must be sure about the number of produced neutrinos number. For a long time, the results of experiments on solar neutrinos (created to detect electron neutrinos) were smaller than predictions. This could be interpreted as a disappearance experiment, but one could question the prediction about the production of solar neutrinos.
- Now we have disappearance experiments with electron anti-neutrinos from nuclear reactors, where we can determine the number of produced neutrinos very precisely.
- We also have appearance experiments on solar neutrinos, in which we reveal not only electron neutrinos but also those of other families.
- Moreover, there are experiments with accelerators and atmospheric neutrinos, which confirm the oscillation phenomenon.

Reactors experiments for anti-ν_e oscillations

- Within reactors electron anti neutrinos are produced with energies of the order of MeV
- The detection occours through charged current reactions anti-ν_e + p -> n + e^+
- Other flavour neutrinos can be produced, but they are "sterile", since the CC reaction anti-ν_μ + p -> n + μ^+ needs higher energies (m_μ=106 MeV)
- So we can perform only disappearence experiments, that means the measurement of the neutrino survival probability, averaged on the neutrino energetic spectrum* :



$$< P_{ee}(L) >= \frac{\int dE \ \frac{d\Phi}{dE} \ P_{ee}(E,L)}{\int dE \ \frac{d\Phi}{dE}}$$

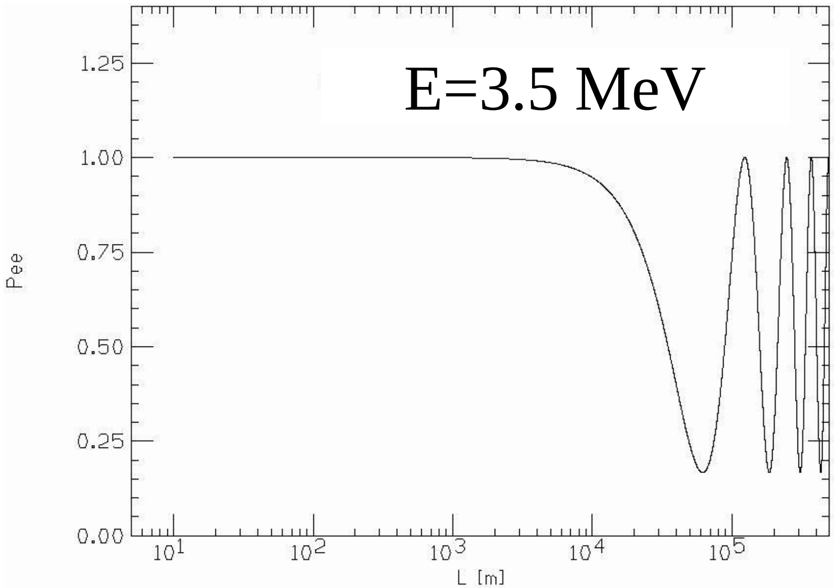
$$P_{ee}(E,L) = 1 - \sin^2 2\theta \ \ \sin^2[\Delta m^2 L/(4E)]$$

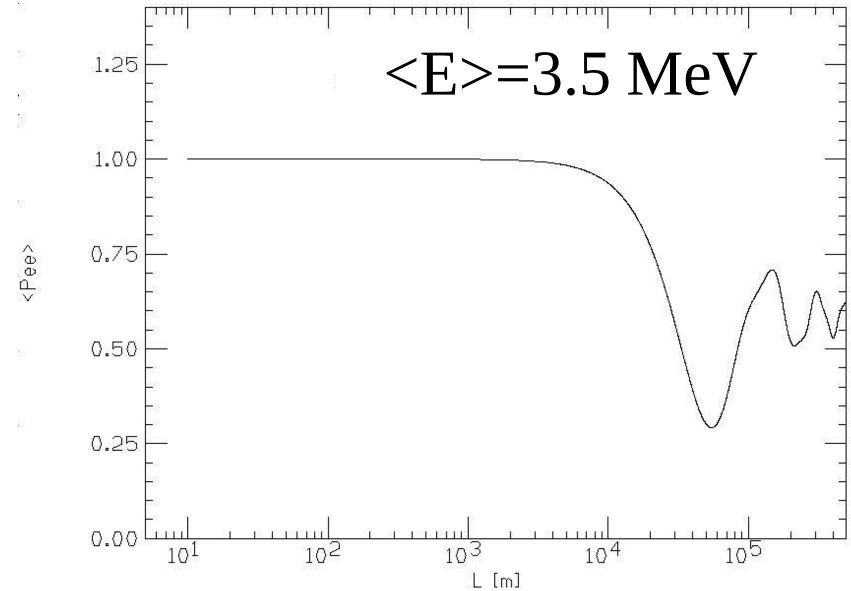
** The probability is averaged also on cross-sections*

Probability and mean probability

$$P_{ee}(E,L) = 1 - \sin^2 2\theta \ \ \sin^2[\Delta m^2 L/(4E)]$$

- The survival probability at a given energy and that averaged on a reactor energy spectrum, are similar, but there are also some differences. The two plots show the situation corresponding to $\Delta m^2 = 7\ 10^{-5}$ eV^2 and $\sin^2 2\theta = 0.8$ (these parameters are chosen for reasons we'll understand later).

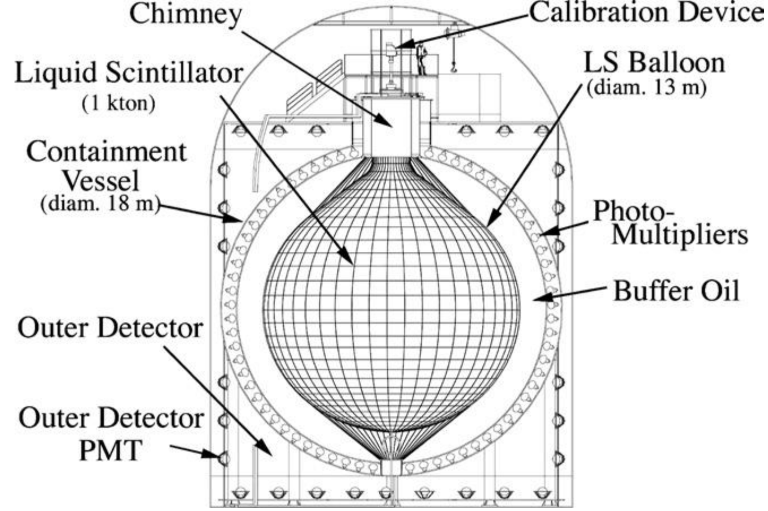




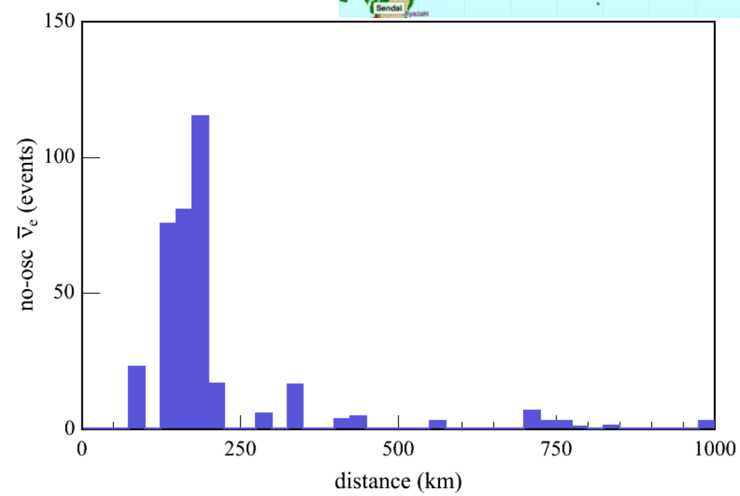
- If the energy is fixed, the survival probability P_{ee} oscillates between 1 and $1-\sin^2 2\theta$
- If we have many energies over which we must average, the oscillating term containing the energy goes to ½, so that the behaviour of $\langle P_{ee} \rangle$ is:
  1) at "small" distances : $\langle P_{ee} \rangle = 1$
  2) at "large" distancies: $\langle P_{ee} \rangle$ 1- 1/2 $\sin^2 2\theta$

  where "small" and "large" are referred to L_v, computed at the mean energy of produced neutrinos: if $\langle E \rangle$ = 3.5 MeV then $L_v \approx 10^5$ m

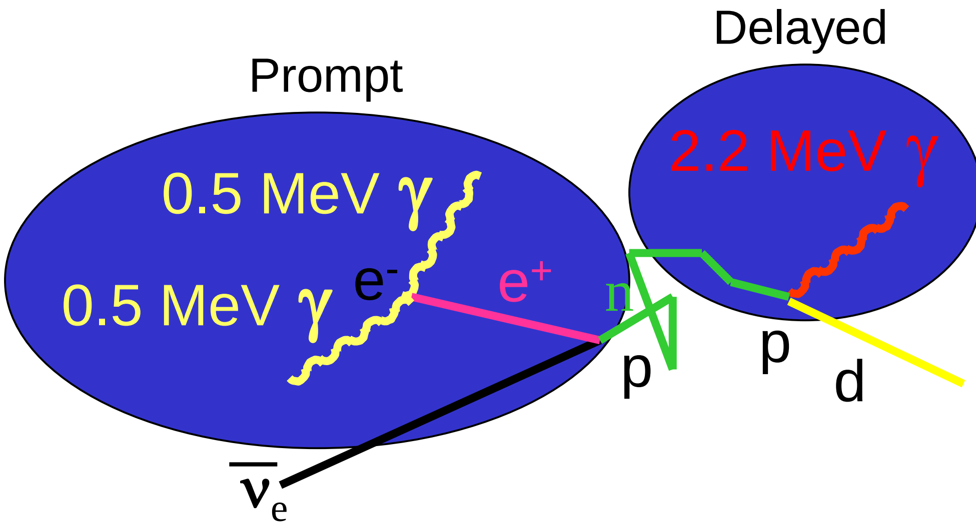
KamLAND, the proof of reactor anti-neutrinos oscillation



- The main parameter for a reactor experiment is the distance between detector and reactor
- Since the neutrino flux scales as $1/L^2$, long distances experiments need large detectors.
- Over several, years experiments at distances of hyndreds of meters (also one kilometer from reactors) were performed, always with negative results.
- The key was KamLAND, a detector with 1000 tons of liquid scintillator, with about one thousand photomultipliers.
- Kamland is surrounded by many reactors, so that the total flux turns out to be about 10^5 cm^2/s
- The mean distance from reactors (weighted on flux) is about 180 km
- We can measure the energy released by charged particles (as electrons and positrons) and that from gamma ray, and we obtain about 300 photo-electrons for each MeV given to the detector.



Anti-neutrino detection method



- The classical process is the inverse beta reaction

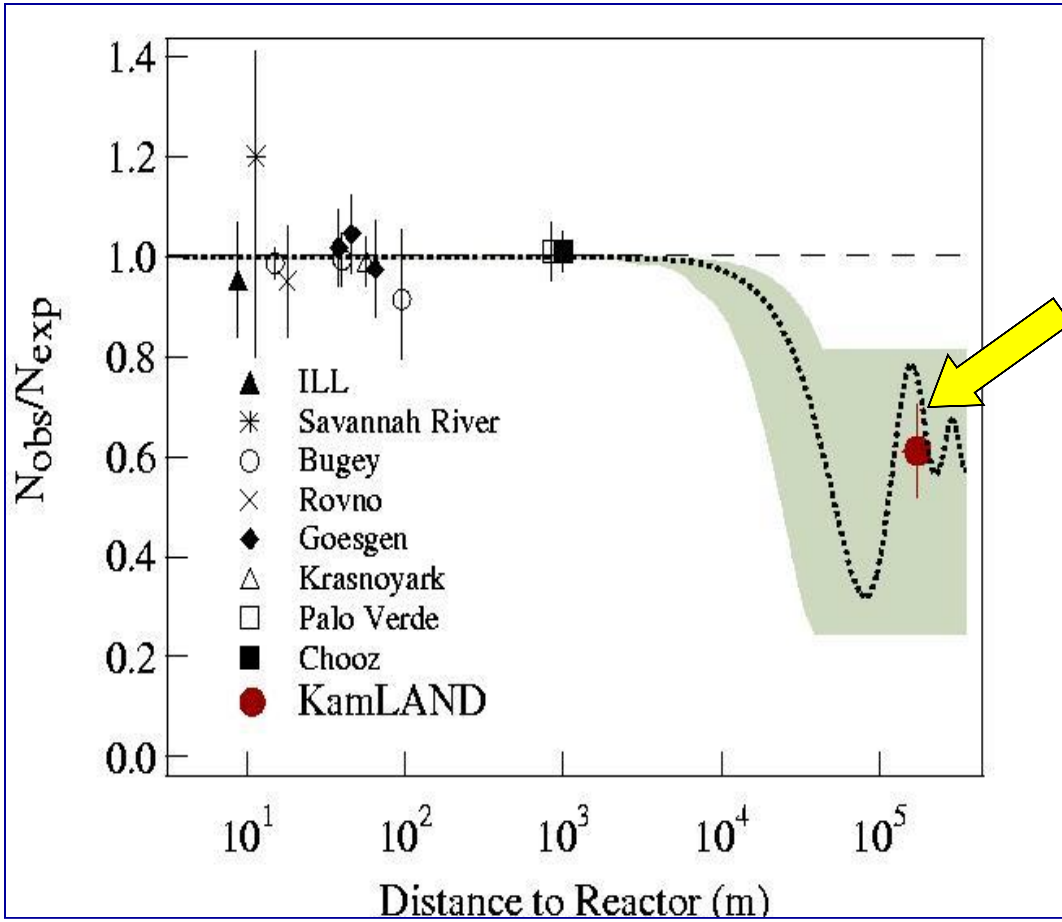
$$\overline{\nu}_e + p \rightarrow e^+ + \mathrm{n}$$

- Target protons are those contained in the organic scintillator, C_nH_{2n},
- The positron looses energy during the slowing down and then it annihilates with an electron.
- The energy which is released is equal to that of netrino, but for the reaction threshold (1.8 MeV), plus the annihilation energy of the positron (1MeV). In total the energy of the prompt signal is equal to that of neutrino, but for 0.8 MeV
- The neutron thermalizes, and after a mean time ~200μs it is captured by a proton, emitting a gamma ray at 2.2 MeV

KamLAND result:
the proof of neutrino oscillation

- The picture summarizes the results of 50 years of reactor experiments.
- In 2002 KamLAND revealed antiν_e from reactors at L≈200 km
- Only 60% of antiν_e survives during the travel source-detector.
- Neutrinos oscillate:

1) since $\Delta m^2 = m_1^2 - m_2^2 \neq 0$, then, at least for one neutrino $m \neq 0$
2) The family lepton number is not exactly conserved, but it is conserved only on distances which are short with respect to the neutrinos oscillation length.

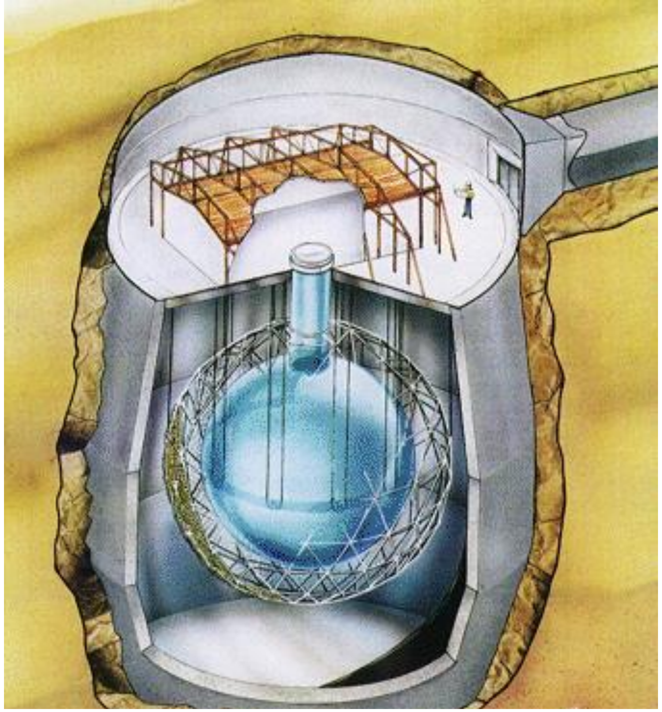


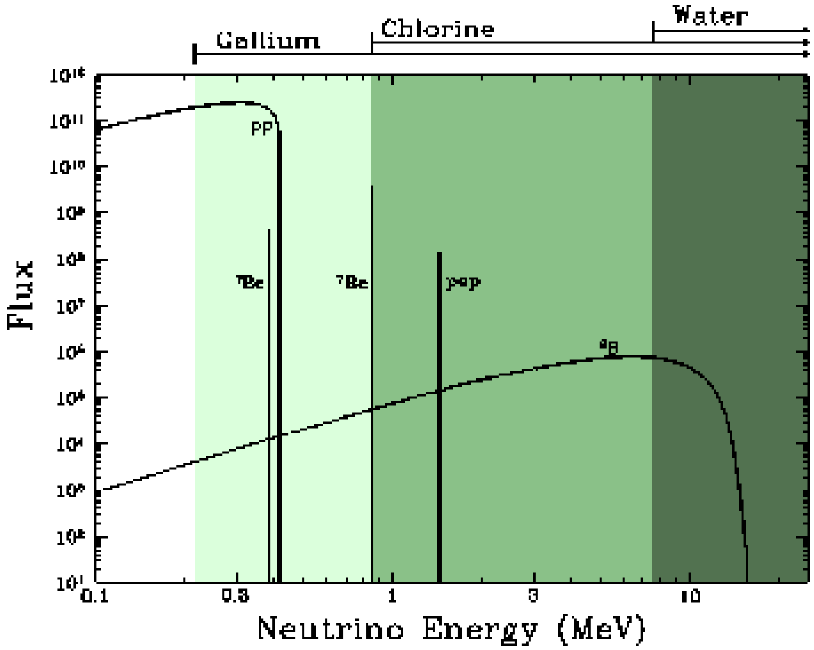
Best fit: $\sin^2 2\theta = 0.92$ [PRL 90 (2003) 021802]

$\Delta m^2 = 7.6 \; x \; 10^{-5} eV^2$

SNO: appearance experiment

- SNO (Sudbury Neutrino Observatory) provided the “smoking gun” for oscillation, since it detected neutrinos with a different flavour from the original one.
- SNO is placed in a Nickel mine in Sudbury, Canada, about 2000 m below ground, corresponding to about 6000 m. w. e. depth.
- It uses 1000 tons of D_20, surrounded about 10.000 phototubes.
- It observes reactions induced by boron solar neutrinos, the most energetic component of neutrinos from the sun.
- The crucial point is that SNO is sensitive both to electron and to other flavours neutrinos.





Reactions induced by neutrinos in SNO

CC $\nu_e + d \rightarrow p + p + e^-$

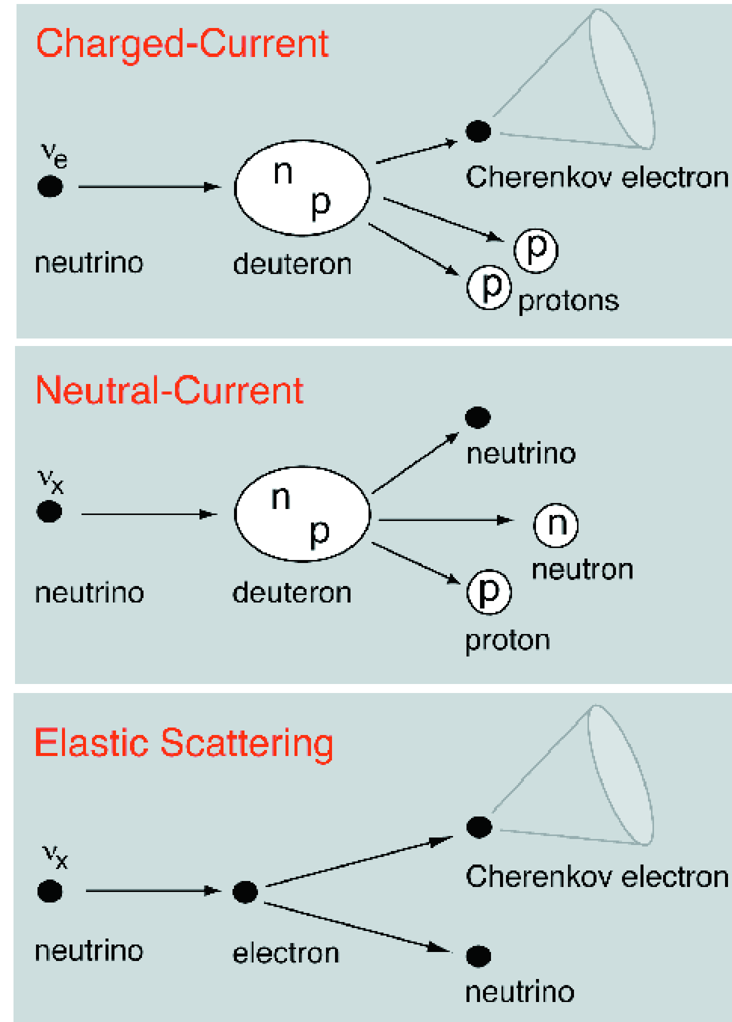
This reaction can only be induced by electron neutrinos. Electrons are diffused almost isotropically. The signal is given by electron Cherenkov radiation.

NC $\nu_x + d \rightarrow p + n + \nu_x$

This reaction can be induced by all kind of neutrinos, with the same cross-section. The signal is now given by the neutron capture.

ES $\nu_x + e^- \rightarrow \nu_x + e^-$

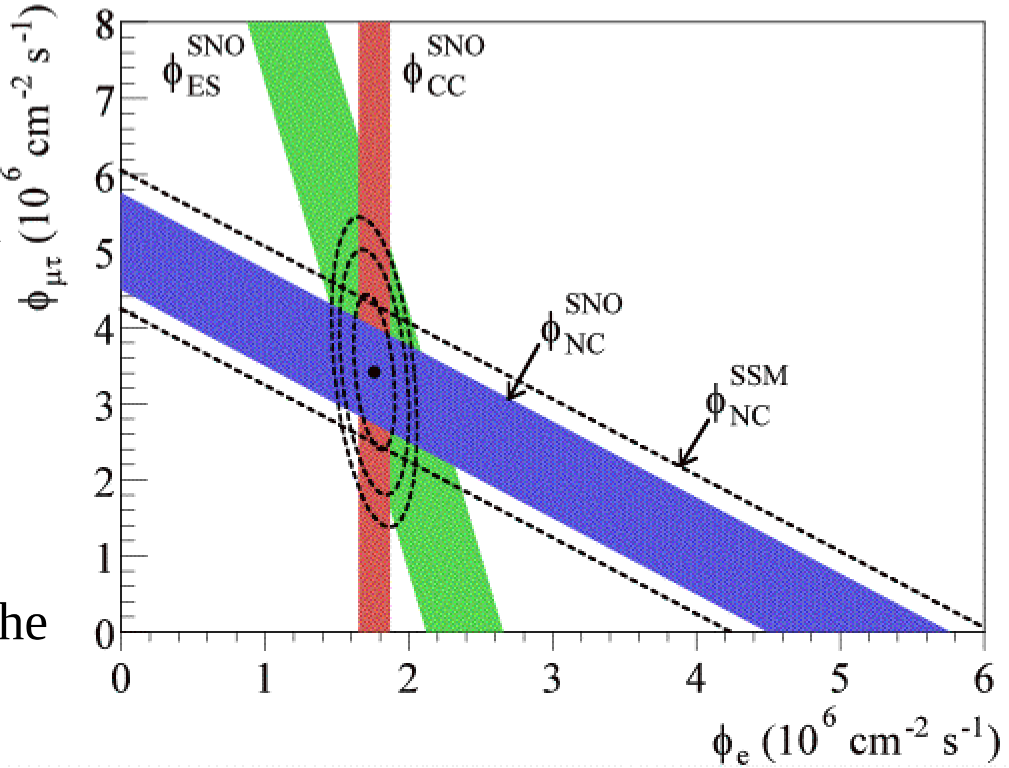
This reaction can be induced by ν_e, but also by other kind of neutrinos, with a smaller cross-section (about 1/6). Electrons are produced in the direction of incoming neutrino, and they are detected by Chernkov radiation.



SNO results

- Results of different measurements can be shown in a plane with electron neutrinos flux on the horizontal axis, while other neutrinos types is on the vertical scale.
- **NC** interactions measure the total flux $\phi=\phi_e+\phi_{\mu\tau}$. Whatever their fate, this is the flux of neutrinos which left the sun as electron ones.
- **CC** interactions only reveal electron neutrinos, and consequently they are described by a vertical band.
- Electron scattering interactions, **ES**, measure an effective flux.

  $\phi=\phi_e+ 1/6\ \ \phi_{\mu t}$ since the $\nu_{\mu,\tau}$ cross.section is 1/6 of that for the ν_e .



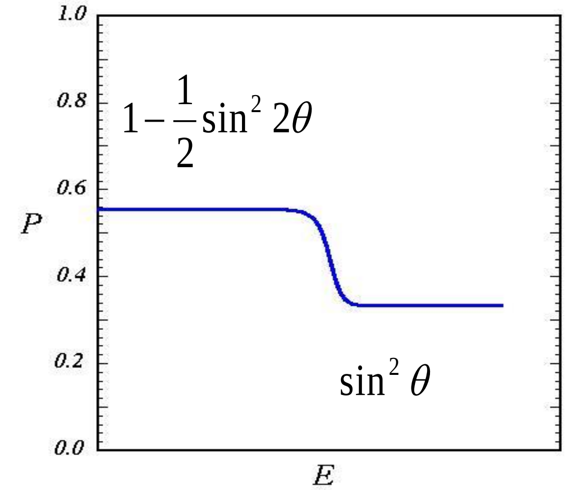
- The important result is that the curves intersect in a point

  $$\phi_{\mu\tau}\neq 0$$

- Results for single fluxes, in the units of the picture, are:

$$\phi_{CC}^{SNO} = 1.67^{+0.05}_{-0.04}(\text{stat})^{+0.07}_{-0.08}(\text{syst})$$
$$\phi_{ES}^{SNO} = 1.77^{+0.24}_{-0.21}(\text{stat})^{+0.09}_{-0.10}(\text{syst})$$
$$\phi_{NC}^{SNO} = 5.54^{+0.33}_{-0.31}(\text{stat})^{+0.36}_{-0.34}(\text{syst}),$$

Survival probability in vacuum and in matter



- We expect that, by weighting over energy, and after a baseline as long as the Earth-Sun distance, the survival probability should reach its asymptotic value:

$$<P_{ee}> = 1 - 1/2 \sin^2 2\theta$$

- We saw with KamLAND that the mixing angle is close to maximum, so we should expect:

$$<P_{ee}> \approx 0.54$$

- On the other hand the SNO result is:

$$<P_{ee}> = \frac{\phi_{CC}^{SNO}}{\phi_{NC}^{SNO}} = 0.301 \pm 0.033(\text{total}).$$

- The two results are not contradictory. The explanation is in the fact that neutrinos crossing the Sun experience interactions with matter, that are different for electron neutrinos and for other kind of neutrinos.
- This alters the mixing angle in matter with respect to that in vacuum. Also the mixing angle becomes dependent on energy. The explaination of such a phenomenon is not in the part of this course, but it is summarized in the appendix for those who are interested.
- The effective survival probabilty is shown in figure.At low energies we get the "vacuum result, while at higher energies, as for Boron , the survival probability is smaller.

Atmospheric neutrinos

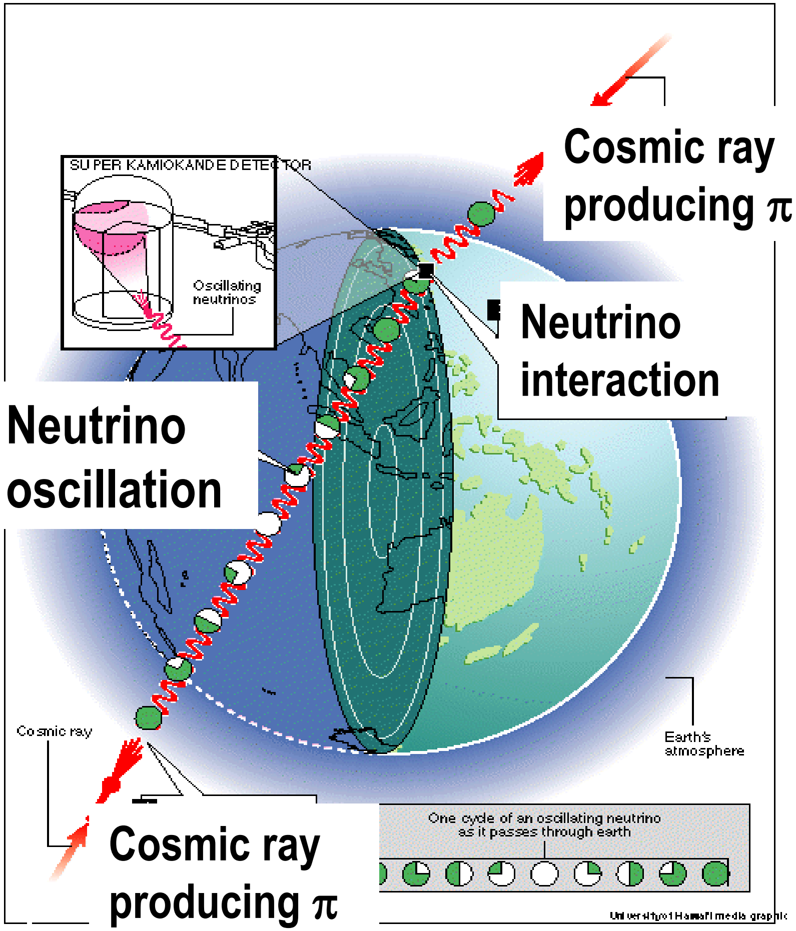
- From interaction of primary cosmic rays (mainly protons) in the high atmosphere, pions are produced, and from their decay also muon and electron neutrinos and their antiparticles

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \qquad \pi^- \rightarrow \mu^- + anti\nu_\mu$$

$$\mu^+ \rightarrow e^+ + \nu_e + anti\nu_\mu \qquad \mu^- \rightarrow e^- + anti\nu_e + \nu_\mu$$

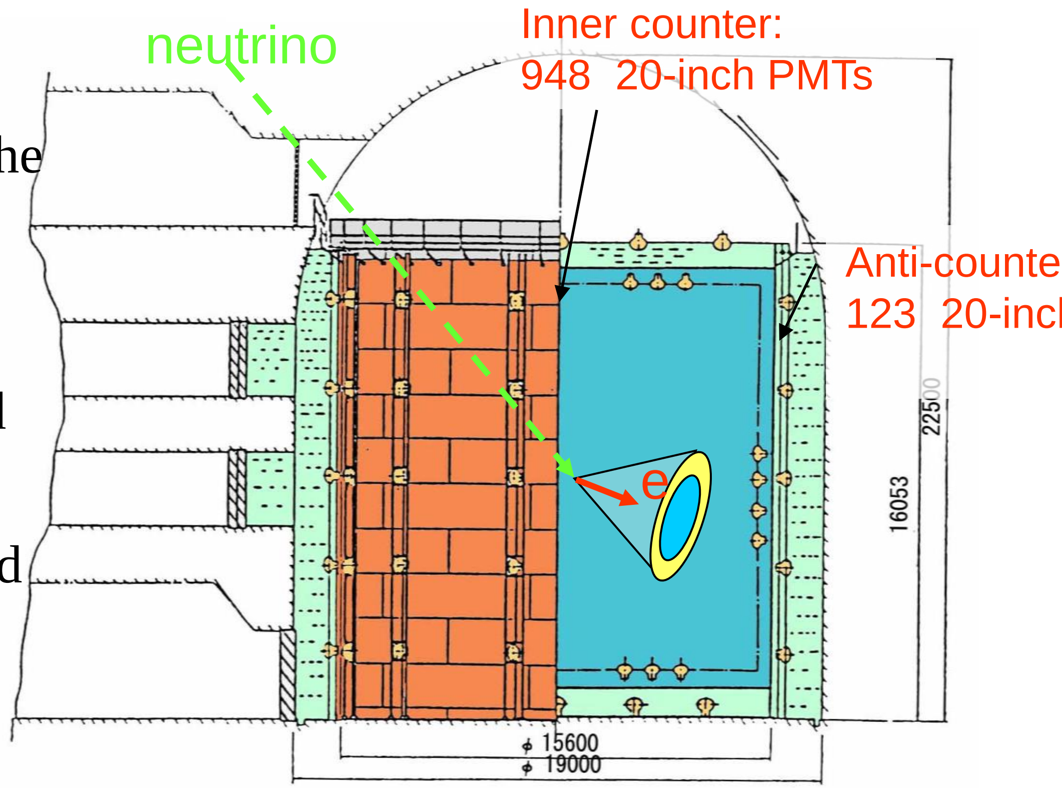
- If the production of π+ and π- are equal, we expect that the number of muon neutrinos and anti-neutrinos is twice that for electrons .

- The energy spectrum of these atmospheric neutrinos is quite wide: from MeV to 10^3 TeV, but with a peak around 1 GeV.



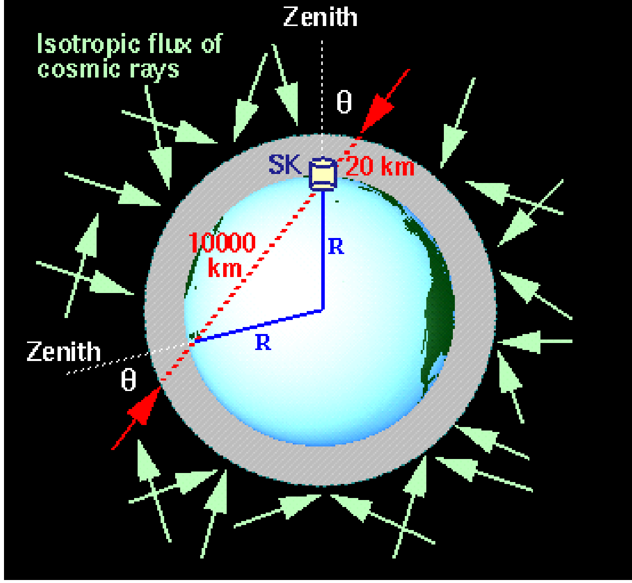
Atmospheric neutrinos: detection (1)

- Kamiokande experiment used several ktons of water to detect the Cherenkov light emitted by charged leptons (electrons or muons) produced by the interaction of neutrinos on H and O nuclei (ν+N->l +X)
- We can distinguish electronic and muonic events.
- The first observed result was an anomalous event ratio with respect to the calculated one.

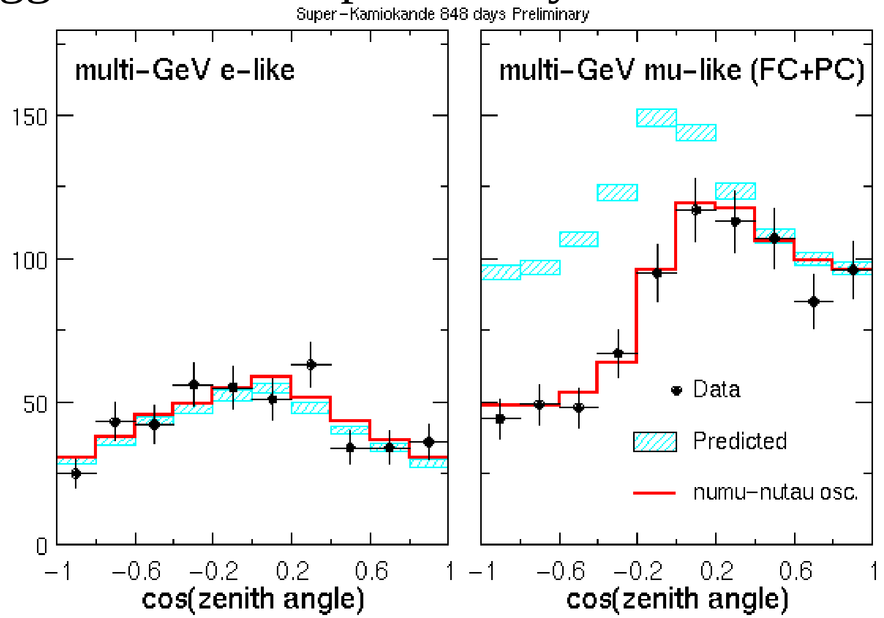


$$R = \left(\frac{\nu_\mu + anti\nu_\mu}{\nu_e + anti\nu_e}\right)_{obs} / \left(\frac{\nu_\mu + anti\nu_\mu}{\nu_e + anti\nu_e}\right)_{cal} = 0.63 +\text{-} 0.04$$

Atmospheric neutrinos: detection(2)



- The Super-Kamiokande experiment employed 50 kTon of water and 11000 phototubes at1000 m depth.
- Super-Kamiokande could compare the number of neutrinos coming from “above” and “below” the detector. These last ones travelled a longer way, since they crossed the whole Earth, and so the have a bigger oscillation probability.



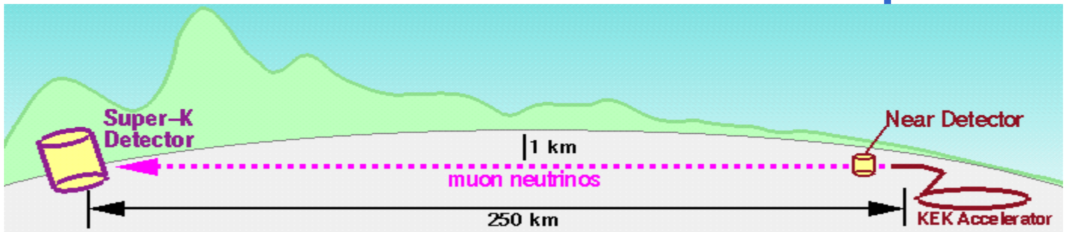
•SuperK observed an azimuthal suppression of the “muonic” signal, while the “electronic” one was NOT suppressed.

•This is consistent with the oscillation $\nu_\mu \to \nu_\tau$ with:

$$\sin^2 2\tilde{\theta} = 1$$

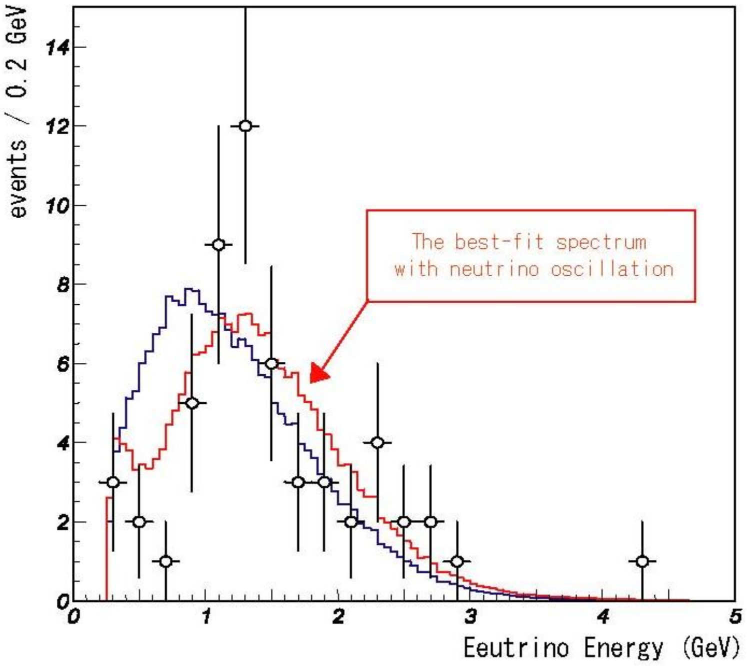
$$\Delta\tilde{m}^2 = 2.2 \times 10^{-3}\, eV^2$$

Accelerator neutrinos: K2K experiment



- Muonic neutrinos beam: “long base” disappearance experiment (d=250 Km)
- Close detector :1 Kton H2O: it measures the number of events and the spectrum before the oscillation
- Far detector: 50 Kton, it reveals the decreased number of events and the distorsion of the spectrum
- Oscillation $\nu_\mu \to ?$ with:

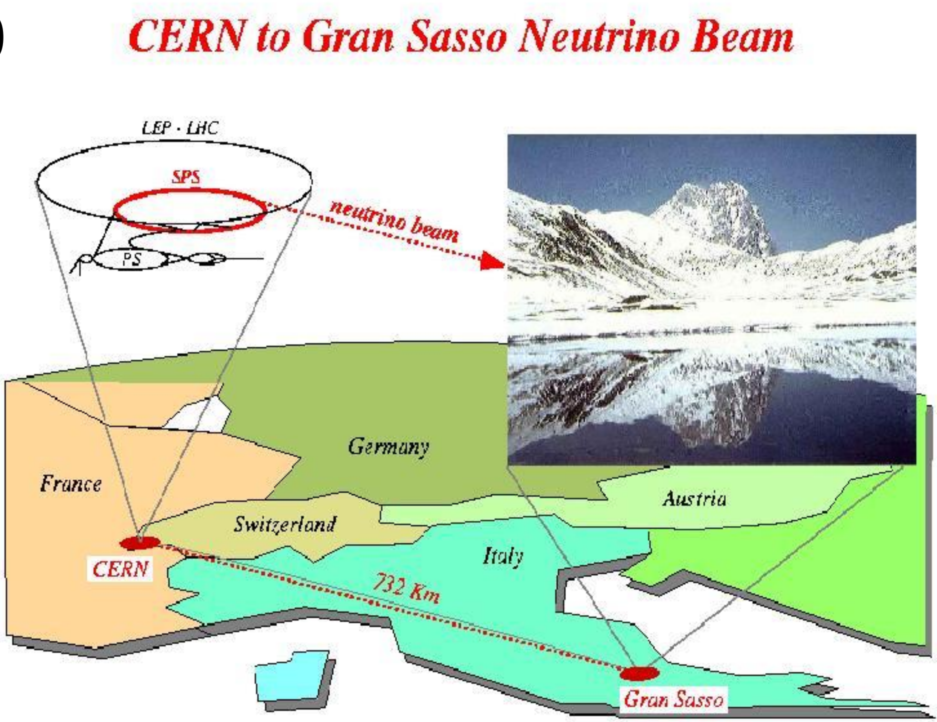
$$\sin^2 2\tilde{\theta} = 1 \qquad \Delta\tilde{m}^2 = 2.8 \ x \ 10^{-3} eV^2$$



- It is consistent with atmospheric neutrinos results on $\nu_\mu \to \nu_\tau$

From Cern to Gran Sasso

- ν_μ produced at CERN (Geneva) are shoot towards the Gran Sasso .
- In their travel of about 730 km, ν_μ can oscillate into ν_τ.
- The Charged Current interaction of ν_τ with matter produce the charged lepton τ, which can be detected through its decay processes
- The Opera experiment at LNGS has detected these tau leptons, confirming the hypothesis of ν_μ - ν_τ oscillation.



Oscillations picture

- Experiments at reactors have shown that anti-ν_e oscillate to other kind of neutrinos. Oscillation of ν_e have been seen in solar neutrino esperiments. This oscillation phenomenon corresponds to a difference in squared masses :

$$\Delta m^2 \approx 7\ 10^{-5} eV^2$$

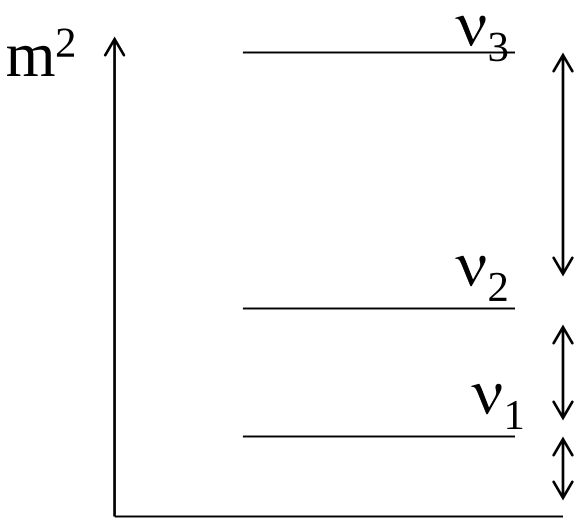
- From atmospheric neutrinos and long-base accelerator experiments, one has seen another oscillation phenomenon , that of ν_μ into ν_τ . In this case the difference in the squared masses is of the order of:

$$\Delta \tilde{m}^2 \approx 2\ \ 10^{-3} eV^2$$

- Note that these processes are different ones.

Neutrino masses picture

- The existence of two different oscillation processes tells us that we have at least two mass eigenstates with non zero mass



$$\Delta\tilde{m}^2 = m_3^2 - m_2^2 \approx 10^{-3} eV^2$$

$$\Delta m^2 = m_2^2 - m_1^2 \approx 7\ 10^{-5} eV^2$$

$$m_1^2 = ?$$

- The absolute mass scale (m_1^2) is unknown.
- Experiments on direct measurements of anti-ν_e mass from tritium decay ($m_{\nu e}^2 < 10\ eV^2$) give information on all m_i:

$$m_{\nu_e}^2 = A(\theta)\ m_1^2 + B(\theta)\ m_2^2 < 10 eV^2$$

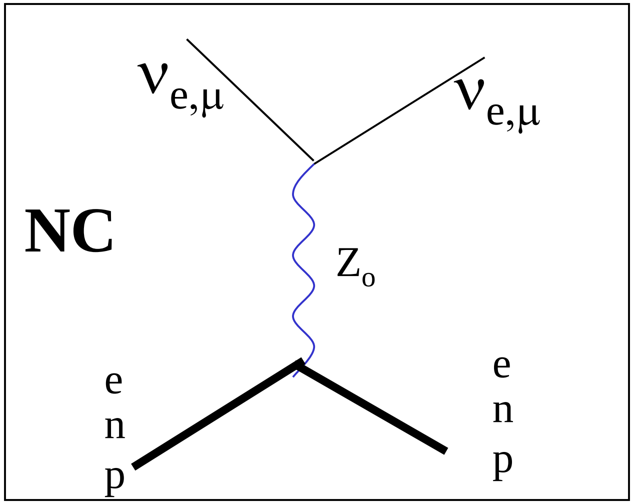
$$m_{1,2}^2 < 10\ eV^2 \pm O(10^{-4} eV^2) \qquad m_3^2 < 10\ eV^2 \pm O(10^{-3} eV^2)$$

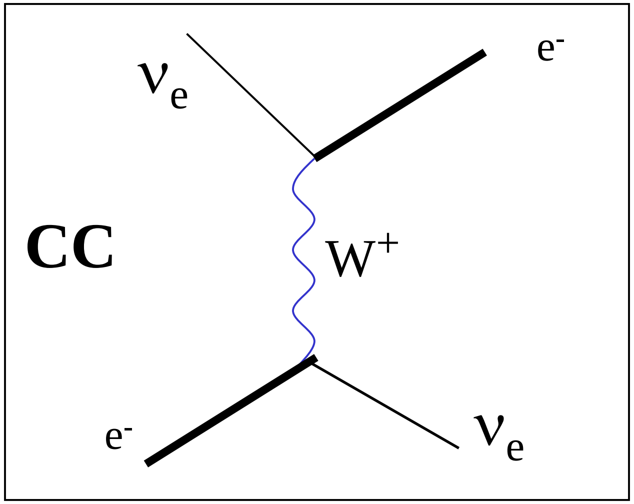
Appendix

- Neutrino propagation in matter

Neutrino propagation in matter

- Up to now we only considered neutrinos propagating in vacuum. But, for instance, solar neutrinos, before being detected, must cross solar and terrestrial matter.
- Neutrino interactions with matter occur through neutral current interaction NC (Z_o exchange) and charged current interactions CC (W^{+-} exchange).





- REMARK:
  - 1) NC interaction produces elastic scatterings ν+ N$\rightarrow\nu$+N, where N=p,n not mediated by CC interactions.
  - 2) NC interaction <u>cannot</u> distinguish flavour, namely the scattering amplitude is the same for any neutrino type.
  - 3) In ordinary matter (e,n,p) <u>only</u> electron neutrino can undergo charged current interactions with matter.

Matter interaction Hamiltonian (1)

- The Hamiltonian describing the coherent interaction with ordinary matter will contain neutral current interaction terms (NC) and those of charged current interaction (CC). In the flavour basis we can write:

$$H_m = \begin{pmatrix} V_{CC} + U_{NC} & 0 \\ 0 & U_{NC} \end{pmatrix}$$

- Since neutral current interaction does not depend on flavour, U_{CN} contributes to H_m with a multiple of the identity operator. It only introduces some phase factors in the time evolution of a flavour state and so it is unrelevant .

- So we only have to compute the charged current contribution.

Some calculations on CC interaction potential

$\hbar = c = 1$

- Let's remember that the EM interaction potential between two electrons is V(r)=e^2/r, so that the interaction energy with a charge distribution with number density n_e will be:

$$V_\gamma(r) \cong \int d^3r' \frac{e^2}{|r-r'|} n_e(r')$$

- CC interaction energy of a neutrino with an electron distribution n_e is like:

$$V_{CC}(r) \cong \int d^3r' \frac{e^2}{|r-r'|} n_e(r') \; e^{-|r-r'|/r_W} \qquad r_w = 1/M_w$$

- For low energy neutrinos (<M_w) we are in the contact interaction approximation [$r_w \rightarrow 0$] and the above expression becomes:

$$V_{CC} \cong e^2 r_W^2 n_e = \frac{e^2}{m_W^2} n_e \cong G_F n_e$$

Where G_F=Fermi constant≈10^{-5} Gev $^{-2}$

- In other words, since weak inetaction (in the low energy limit) is a contact interaction, the neutrino "feels" the charge distribution where it is. In the EM interaction, at the contrary, the electron feels also the distant charges, since the EM force is a long range interaction.

- A more precise calculation (due to Wolfenstein in 1978) brought to:

$$V_{CC} = \sqrt{2} G_F n_e$$

Matter interaction Hamiltonian (2)

- At this point we are able to write the Hamiltonian of the coherent interaction with ordinary matter, in the flavour basis:

$$H_m = \begin{pmatrix} V_{CC} + U_{CN} & 0 \\ 0 & U_{CN} \end{pmatrix} = \begin{pmatrix} V_{CC} & 0 \\ 0 & 0 \end{pmatrix} + U_{CN} Id$$

$$= \begin{pmatrix} \sqrt{2} G_F n_e & 0 \\ 0 & 0 \end{pmatrix} + \ldots = \frac{1}{2} \begin{pmatrix} \sqrt{2} G_F n_e & 0 \\ 0 & -\sqrt{2} G_F n_e \end{pmatrix} + \ldots$$

- Where +… means that we neglect terms which are multiples of the identity operator, and in order to obtain the last passage we added a term $-\sqrt{2}\, G_F n_e$
- Note that the measurement units of the last term are $[\ E^{-2}\ E^3\ ] = [\ E\ ]$ in the natural units system, since the hamiltonian is an energy.
- We can define a refraction length as $L_{rif} = \dfrac{2\pi(\hbar c)}{\sqrt{2} G_F n_e} \cong 10^4\, Km \left(\dfrac{10^{24}\, cm^{-3}}{n_e} \right)$
- Let's remember that when we dealt with oscillation in vacuum we introduced a vacuum oscillation length

$$L_\nu = \frac{4\pi E(\hbar c)}{\Delta m^2} = 2.5 m \left(\frac{eV^2}{\Delta m^2} \right) \left(\frac{E}{MeV} \right)$$

- The comparison between these two lengths will decide how neutrinos propagate in matter

Vacuum Hamiltonian in flavour basis

$\hbar = c = 1$

- Let's remember the relation connection between the two bases:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

θ=vacuum mixing angle

$$\Delta m^2 = m_2^2 - m_1^2 > 0$$

- We can write the vacuum Hamiltonian in the basis (ν_e, ν_μ), but for multiples of the identity operator:

$$H_\nu = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} -\frac{\Delta m^2}{4E} & 0 \\ 0 & \frac{\Delta m^2}{4E} \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}$$

Total Hamiltonian= vacuum+matter

- The total hamiltonian describing neutrino propagation in the flavour basis is given by, but for multiples of identity operator:

$$H_{tot} = H_v + H_m = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \frac{\sqrt{2}}{2}G_F n_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta - \frac{\sqrt{2}}{2}G_F n_e \end{pmatrix}$$

- Note the two contributions: 1) mass $\frac{\Delta m^2}{4E}\cos 2\theta$ and 2) matter $\frac{\sqrt{2}G_F n_e}{2}$

When $\frac{\Delta m^2}{4E} > \frac{\sqrt{2}G_F n_e}{2}$, so $L_V < L_M$ we say that the oscillation is dominated by masses, and we are in the case of vacuum oscillation. In the opposite case we say that the oscillation in dominated by matter.

- The flavour states evolution is consequently given by:

$$i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = H_{tot}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

- In general,to find the oscillation probability, we should integrate numerically the evolution equation. The case of constant matter density is simpler…...

Mixing angle in matter

$\hbar=c=1$

- The full Hamiltonian is not diagonal neither in the mass basis, nor in the flavour one.
- However it can be diagonalized in a new basis, called matter eigenstates and indicated as $|\nu_{i\,m}>$.
- In order to do this we should make a rotation with mixing angle θ_m.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} \begin{pmatrix} \nu_{1_m} \\ \nu_{2\,m} \end{pmatrix}$$

θ_m= mixing angle in matter

- It can be demonstrated that the mixing angle is given by:

$$\sin\ 2\theta_m = \frac{\left(\frac{\Delta m^2}{4E}\right)\sin 2\theta}{\sqrt{\left(\frac{\Delta m^2}{4E}\right)^2 \sin^2 2\theta + \left(\frac{\Delta m^2}{4E}\cos 2\theta - \frac{\sqrt{2}G_F n_e}{2}\right)^2}}$$

and that the separation between the two eigenvalues is $\Delta^2/2E$ where:

$$\Delta^2 = \Delta m^2 \sin 2\theta / \sin 2\theta_m$$

Concluding, we have just found the same formalism of vacuum oscillations but with:

$\Delta m^2 \rightarrow \Delta^2$ e $\theta \rightarrow \theta_m$

Oscillation probability in matter with constant density

- So, the probability that an electron neutrino with E energy is transformed into another flavour, after travelling a distance L in ordinary matter with constant density is:

$$P_{e\mu} = \sin^2 2\theta_m \quad \sin^2[\frac{\Delta^2}{4E}L]$$

$$= \frac{\left(\frac{\Delta m^2}{4E}\right)^2 \sin^2 2\theta}{\left(\frac{\Delta m^2}{4E}\right)^2 \sin^2 2\theta + \left(\frac{\Delta m^2}{4E}\cos 2\theta - \frac{\sqrt{2}G_F n_e}{2}\right)^2} \sin^2\left(\sqrt{\left(\frac{\Delta m^2}{4E}\right)^2 \sin^2 2\theta + \left(\frac{\Delta m^2}{4E}\cos 2\theta - \frac{\sqrt{2}G_F n_e}{2}\right)^2} \quad L\right)$$

NOTE:

- 1) Maximum oscillation amplitude if: $\frac{\Delta m^2}{4E}\cos 2\theta = \frac{\sqrt{2}G_F n_e}{2}$ $(L_\nu = L_{rif}\cos 2\theta)$
Existence of RESONANCE: even for small mixing angles θ, we can have large oscillation, the mixing effect being increased by interaction with matter.
- 2) There is oscillation if: $\frac{\Delta m^2}{4E}\sin 2\theta\, L \geq 1$ $(L \sin 2\theta \geq L_\nu = L_{rif}\cos 2\theta)$
RESONANCE efficiency: the resonance effect occurs if $L \approx L_{rif}$
- 3) If the matter term is negligible($\frac{\Delta m^2}{4E} >> \sqrt{2}G_F n_e$) we find again vacuum oscillations. (Verify that this occurs for terrestrial neutrinos)
- 4) for fixed density and Δm^2, the suppression depends on the neutrino energy.

Survival probability in the solar interior

- A good appoximation for the survival probability of a solar neutrino, after travelling the entire Sun, if sen^2 $2\theta \approx 1$ and $\Delta m^2 \approx 10^{-4}$ eV^2 is given by:

$$P_{ee} = \frac{1}{2} + \frac{1}{2} \frac{\frac{\Delta m^2}{4E}\cos 2\theta - \frac{\sqrt{2} G_F n_o}{2}}{\sqrt{\left(\frac{\Delta m^2}{4E}\right)^2 \sin^2 2\theta + \left(\frac{\Delta m^2}{4E}\cos 2\theta - \frac{\sqrt{2} G_F n_o}{2}\right)^2}} \cos 2\theta$$

n_o=electronic density in ν_e production point

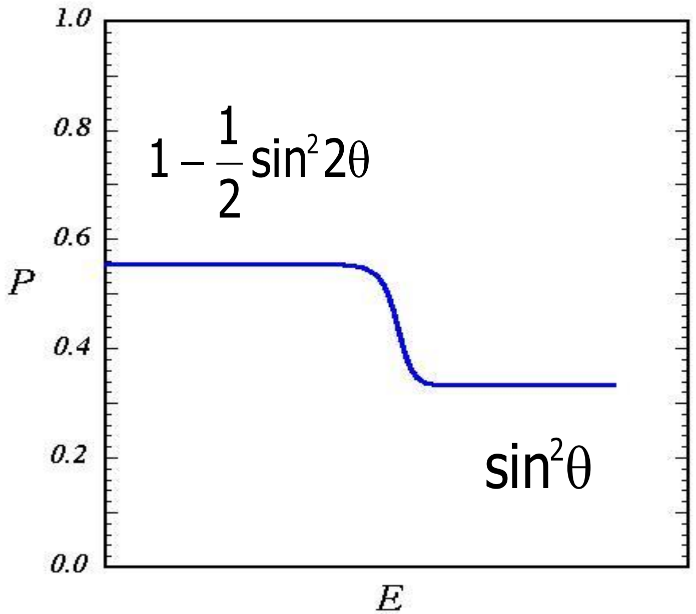
- Two regimes:

1) low E (VACUUM) mass dominant

$$\frac{\Delta m^2}{2E}\cos 2\theta > \frac{\sqrt{2} G_F n_o}{2} \qquad P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta$$

2) high E (MATTER) matter dominant

$$\frac{\sqrt{2} G_F n_o}{2} > \frac{\Delta m^2}{2E} \qquad P_{ee} = \frac{1}{2} - \frac{1}{2}\cos 2\theta = \sin^2 \theta$$



- Due to the conditions of the Sun centre where neutrinos are produced ($n_o=10^{26}$ cm^{-3}) and to $\Delta m^2 \approx 10^{-4}$ eV^2 , the transition between the two regimes occours for MeV energies. So, in the same production region, electronic neutrinos with energies higher than MeV are more suppressed than lower energies ones.